Name: ______________________________

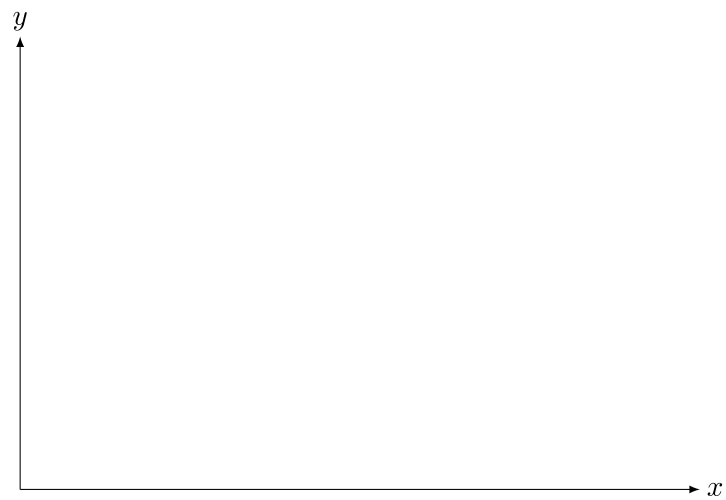
Math 253 Calculus III (Bueler) Wednesday 28 March 2018

Midterm Exam II

In class. 60 minutes. No calculator. $\frac{1}{2}$ sheet of notes allowed. 100 points total.

1. (*10 pts*) Draw a contour map of the function showing several level curves (contours). Specifically, draw three contours in the first quadrant. (That is, assume $x \geq 0$ and $y \geq 0$ as shown.) Label each contour with the corresponding function value.

$$f(x, y) = x + y^2$$



2. (*10 pts*) Find the position vector $\mathbf{r}(t)$ of a particle that has the given acceleration and the given initial velocity and initial position.

$$\mathbf{a}(t) = 2\,\mathbf{i} + 2t\,\mathbf{k}, \quad \mathbf{v}(0) = 3\,\mathbf{i} - \mathbf{j}, \quad \mathbf{r}(0) = \mathbf{j} + \mathbf{k}$$

3. (a) (*5 pts*) Find all the first partial derivatives.

$$f(x, y) = \ln(ax + by)$$

(b) (*10 pts*) Now find all the second partial derivatives. Correct notation is required for full credit.

4. (*10 pts*) Use a tree diagram to write out the chain rule(s) for the given case. Assume all functions are differentiable. Include both a tree diagram and the chain rule(s) in your answer.

$u = f(x, y), \quad \text{where } x = x(r, s, t), \; y = y(r, s, t)$

5. (a) (*5 pts*) Find the gradient of the function.

$$f(x, y, z) = xy^2 \arctan(z)$$

(b) (*10 pts*) Now find the directional derivative of f at the given point in the direction of $\mathbf{v}$.

$$(2, 1, 1), \qquad \mathbf{v} = \langle 1, 1, 1 \rangle$$

6. (a) (*10 pts*) Find the critical point(s) of the function.

$$f(x, y) = x^2 - 2xy + y^3 - \frac{1}{2}y^2$$

(b) (*10 pts*) Find all of the local minimum and maximum values and saddle point(s) of the function in part **(a)**. Clear application of the second derivative test is required.

7. (*10 pts*) Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

$$e^z = xyz$$

8. (*10 pts*) Find an equation of the tangent plane of the given surface at the specified point. Write your equation in the form $ax + by + cz + d = 0$.

$$xy^2z^3 = 8, \quad (2, 2, 1)$$

Extra Credit. (*3 pts*) Give an example of a function $f(x, y)$ which has only one critical point, which is a local minimum, but for which $D = 0$ at the critical point. Then give an example of another function $g(x, y)$ which again has only one critical point, this time a saddle point, where again $D = 0$ at the critical point.

[BLANK SPACE FOR SCRATCH WORK. CLEARLY-LABEL ANYTHING YOU WANT TO BE GRADED.]