VERSION A　　Name: ____________________

Math 253 Calculus III (Bueler)　　Friday 4 May 2018

Final Exam

In class. 120 minutes. No calculator. 1 sheet of notes allowed. 200 points total.

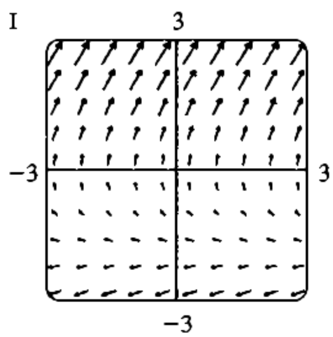
1. (*12 pts*) Match the vector fields $\mathbf{F}$ with the plots labeled I,II,III,IV:

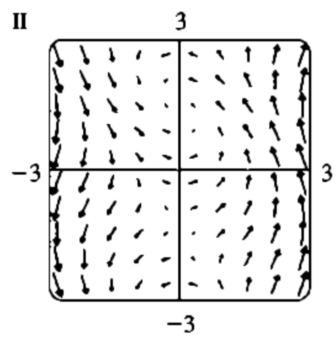
$\mathbf{F}(x,y) = \langle x, -y\rangle$ ______ $\longleftarrow$ *write "I", "II",... in the spaces*

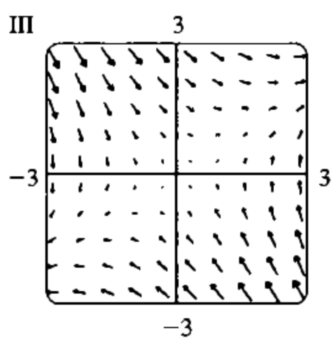
$\mathbf{F}(x,y) = \langle y, x-y\rangle$ ______

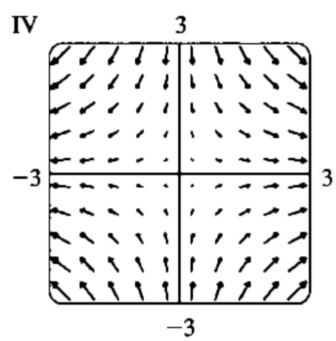
$\mathbf{F}(x,y) = \langle y, y+2\rangle$ ______

$\mathbf{F}(x,y) = \langle \cos(x+y), x\rangle$ ______









2. (*13 pts*) Suppose $\mathbf{F}(x,y) = \left\langle x^2 + y, 3x - y^2\right\rangle$. Calculate $\int_C \mathbf{F}\cdot d\mathbf{r}$ for *any* positively-oriented, closed, simple curve C enclosing a region D which has area 6. (*Hint.* Green's Theorem)

3. (*15 pts*) Evaluate the line integral if C is the line segment from $(2,0)$ to $(5,4)$:

$$\int_C xe^y\, ds$$

4. (*15 pts*) Determine whether $\mathbf{F}$ is a conservative vector field. If it is, find f so that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x,y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$$

5. (*15 pts*) The solid E is the tetrahedron bounded by the planes $x = 0, y = 0, z = 0, x+y+z = 1$. Find the mass of the solid E if its density is $\rho(x, y, z) = 2x$.

6. (*10 pts*) Evaluate the double integral if D is bounded by $y = 0$, $y = x^2$, and $x = 1$:

$$\iint_D x \cos y \, dA$$

7. **(a)** (*10 pts*) Sketch the solid which is above the cone $z = \sqrt{x^2 + y^2}$ and inside (below) the sphere $x^2 + y^2 + z^2 = 1$.

(b) (*10 pts*) Use *spherical coordinates* to find the volume of the solid in part **(a)**

8. (*10 pts*) Find a unit vector that is orthogonal to both $\mathbf{i}+\mathbf{j}$ and $\mathbf{i}+\mathbf{k}$.

9. (*15 pts*) Find an equation of the plane through the point $(1,-1,-1)$ and parallel to the plane $5x-y-z=6$. Simplify to the form $ax+by+cz+d=0$.

10. (*10 pts*) Set up, but do not evaluate, an integral to find the length of the curve:

$$\mathbf{r}(t)=\left\langle t^2, t^3, t^4\right\rangle, \qquad 1 \le t \le 5$$

11. **(a)** (*10 pts*) Sketch the plane curve C with the given vector equation. Label axes and show the orientation of the curve.

$$\mathbf{r}(t) = 4\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}, \qquad 0 \le t \le \frac{\pi}{2}$$

(b) (*10 pts*) Compute the unit tangent vector $\mathbf{T}(t)$ for the curve C in part `(a)`.

12. (*10 pts*) Find and simplify the linearization $L(x, y)$ of the function at the point:

$$f(x, y) = \sqrt{xy}, \qquad (1, 4)$$

13. (*15 pts*) Suppose

$$w = xy + yz + zx, \qquad x = r\cos\theta, \quad y = r\sin\theta, \quad z = r\theta$$

Compute the partial derivative $\frac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \pi/2$.

14. **(a)** (*10 pts*) Find all critical points of the function:
$f(x, y) = 2 - x^4 + 2x^2 - y^2$

(b) (*10 pts*) Find all local maxima, local minima, and saddle points of the function in part **(a)**.

Extra Credit. (*5 pts*) Assume $\mathbf{F}(x, y)$ is a conservative vector field defined on a open, connected region D. Fix any point (a, b) in D. Explain why the formula

$$f(x, y) = \int_{(a,b)}^{(x,y)} \mathbf{F} \cdot d\mathbf{r}$$

defines a function f on D. Explain why this function satisfies $\nabla f = \mathbf{F}$.

[Blank space for scratch work. Clearly-label anything you want to be graded.]