

Name: _____

Math 253 Calculus III (Bueler)

Wednesday 14 February 2018

Midterm Exam I

In class. 60 minutes. No calculator. $\frac{1}{2}$ sheet of notes allowed. 100 points total.

1. (10 pts) Find the area of the triangle PQR :

$$P(0, 0, -3), \quad Q(4, 2, 0), \quad R(1, 1, -1)$$

2. (10 pts) Find an equation of the plane which is through the point $(1, -1, -1)$ and parallel to the plane $5x - y - z = 6$. Simplify the equation to the form “ $ax + by + cz + d = 0$.”

3. (10 pts) Show that the equation represents a sphere, and find its center and radius:

$$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$$

4. (10 pts) Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

5. (10 pts) Determine the angle between the vectors. (*Your answer should be a concrete expression, but it does not need to be simplified.*)

$$\mathbf{a} = \langle 4, 5, -2 \rangle, \quad \mathbf{b} = \langle 3, -1, 5 \rangle$$

6. Assume $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ are vectors. Are the following expressions meaningful or not? The first one is done for you.

	$\mathbf{a} \times \mathbf{b} + \mathbf{c}$	<input checked="" type="checkbox"/> meaningful	<input type="checkbox"/> not meaningful
(a) (2 pts)	$(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$	<input type="checkbox"/> meaningful	<input type="checkbox"/> not meaningful
(b) (2 pts)	$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$	<input type="checkbox"/> meaningful	<input type="checkbox"/> not meaningful
(c) (2 pts)	$ \mathbf{a} (\mathbf{b} \cdot \mathbf{c})$	<input type="checkbox"/> meaningful	<input type="checkbox"/> not meaningful
(d) (2 pts)	$\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$	<input type="checkbox"/> meaningful	<input type="checkbox"/> not meaningful
(e) (2 pts)	$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$	<input type="checkbox"/> meaningful	<input type="checkbox"/> not meaningful

7. (10 pts) Use the space at right to sketch the curve which has the given vector equation. Indicate with an arrow the direction in which t increases. Be sure to (at least) label and give a scale to each axis.

$$\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + \mathbf{k}$$

8. (10 pts) Find a vector equation, *and* parametric equations, of the line through the points $(0, \frac{1}{2}, 1)$ and $(2, 1, -3)$.

Extra Credit. (3 pts) Consider a curve $\mathbf{r}(t)$ for $a \leq t \leq b$. Recall that we have formulas for the unit tangent vector \mathbf{T} and for the curvature κ . Also recall that the arclength function is $s(t) = \int_a^t |\mathbf{r}'(u)| du$ and the unit normal vector is $\mathbf{N}(t) = \mathbf{T}'(t)/|\mathbf{T}'(t)|$. Show that $\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}$.

9. Consider the curve $\mathbf{r}(t) = \langle -1, \frac{1}{3}t^3, \frac{1}{2}t^2 \rangle$ for $0 \leq t \leq 2$.

(a) (10 pts) Compute the arclength.

(b) (5 pts) Calculate (and simplify) the unit tangent vector $\mathbf{T}(t)$.

(c) (5 pts) Calculate (and simplify) the curvature $\kappa = \kappa(t)$.

[BLANK SPACE FOR SCRATCH WORK. CLEARLY-LABEL ANYTHING YOU WANT TO BE GRADED.]