MATH 253UX1 - Final Exam

Mahmud Alam - Fall 2020

Wednesday, December 09, 2020

Print Your First & Last Name CLEARLY	Date

INSTRUCTIONS:

- You must work on your own. You may not get help from any other person.
- Calculator is not allowed.
- This exam is **8 pages** long, not including this cover sheet.
- Once you access this exam in **gradescope** you will have **2 hours** to submit your solutions.
- Download the test and print it out. Write your solutions on the test paper. When you are done, scan the pages and save the document as a **PDF** file. Upload the file back into **gradescope**.
- If you do not have a printer, you may write your answers on blank paper. But use a total of 9 pages leaving the first-page blank as a cover sheet. **Find that some problems are written on a single page. You must write the answer to those problems using a single page, and if there are two problems on a single page, you must answer those problems on a single page.** Finally, make sure the problems are clearly labeled and in order. Do not submit any additional pages, and do not leave any blank pages.
- In order to receive full credit you must show your work. Include your computations on the exam paper.
- Email me immediately if you have any trouble submitting your completed exam. You can also email me if you want to check that your exam uploaded correctly.

Total Possible Points	Score	Percent
100		

Problem 1: Suppose that the temperature T, in degree Celsius, in a certain region of space is given by the function

$$T(x,y,z) = 5x^2 - 3xy + yz,$$

where the position coordinates are in meters.

a) (5 points) What is the directional derivative at $(2,0,3)$ in the direction towards $(3,-2,3)$? Indicate units for your answer.

b) (4 points) From the point $(2,0,3)$, in what direction should you begin moving to experience the greatest rate of cooling, and what would the rate be?

Problem 2: (5 Bonus points) Show that the Cobb-Douglas production function $P = bL^{\alpha}K^{\beta}$ satisfies the equation

$$L\frac{\partial P}{\partial L} + K\frac{\partial P}{\partial K} = (\alpha+\beta)P.$$

Problem 3: (10 points) Use the transformation $x = 2u + v, y = u + 2v$ to evaluate the integral $\iint_R (2x + y)dA$, where R is the triangle with vertices at $(0, 0)$, $(2, 1)$ and $(1, 2)$.

Problem 4: (7 points) Find the area of the surface of the helicoid with vector equation $\mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle$, $0 \leq u \leq 1$ and $0 \leq v \leq \pi$.

Problem 5: A surface S is parameterized by $\mathbf{r}(u, v) = \langle u^2, u + v, u - v^2 \rangle$, $0 \le u \le 2$, $0 \le v \le 4$

a) (4 points) Find an equation for the tangent plane to the surface at the point given by $u = 1, v = 2$.

b) (6 points) Give an integral that would compute the flux of the vector field $\mathbf{F} = \langle 0, x, -y \rangle$ through S, oriented so that the normal vector has a positive z-component.

Problem 6: Consider a particle is moving through space along the curve C given by the following parametric representation $\mathbf{r} = \langle t^3 - 3t + 1, \frac{t}{2} + 1, \frac{t}{2} cos\pi t \rangle, 0 \leq t \leq 2$ and subject to the vector field: $\mathbf{F}(x, y, z) = \langle y^3 - 2xz, 3xy^2 + 2z, 2y - x^2 \rangle$.

a) (3 points) Show that **F** is conservative.

b) (4 points) Find all potential functions for the field **F**

c) (3 points) Use the Fundamental Theorem of Line Integrals to compute the work done on the particle.

d) (8 points) Using the same initial and final points, find a simpler path parallel to the axes between them and then compute the work done again.

<u>Problem 7:</u> (10 points) Verify Green's theorem to evaluate $\oint\limits_C x^2y^2dx + xydy$, C consists of the arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$ and the line segment $(1, 1)$ to $(0, 1)$ and from $(0, 1)$ to $(0, 0)$.

Problem 8: Let S be the closed surface whose bottom is the cone $z = \sqrt{x^2 + y^2}$ and whose top is the plane $z = 4$, oriented outward.

a) (7 points) Set up an integral to find the volume V of the solid Q bounded by the surface S, and evaluate the integral.

b) (5 points) Use Gauss's Divergence Theorem to compute the flux of the field $\mathbf{F} = \langle x + yz, x^2 + z^2, x^2 + z \rangle$.

Problem 9: (6 points) Set up (but do not evaluate) the triple integral $\iiint_Q 4yz dV$ in spherical coordinates where Q is the solid inside the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 1$.

Problem 10: (10 points) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle 1, x + yz, xy - \sqrt{z} \rangle$, and C is the positively oriented boundary of the part of the plane $3x + 2y + z = 1$ in the first octant.

Problem 11: (8 points) Find the minimum value of $f(x, y, z) = x^2 + 2y^2 + 3z^2$ subject to the constraint $x + 2y + 3z = 10$. Show that f has no maximum value with this constraint.