MATH 253 - Midterm # 2

Mahmud Alam - Fall 2020

Wednesday, October 28th 2020

Print Your First & Last Name CLEARLY	Date

INSTRUCTIONS:

- You must work on your own. You may not get help from any other person.
- Once you access this exam in **gradescope** you will have **2 hours** to submit your solutions.
- Download the test and print it out. Write your solutions on the test paper. When you are done, scan the pages and save the document as a **PDF** file. Upload the file back into **gradescope**.
- If you do not have a printer you may write your answers on separate paper. Make sure the problems are clearly labeled and **in order**. Only write on one side of the page.
- In order to receive full credit you must show your work. Include your computations on the exam paper.
- This exam is **7 pages** long, not including this cover sheet.
- Email me immediately if you have any trouble submitting your completed exam. You can also email me if you want to check that your exam uploaded correctly.

Total Possible Points	Score	Percent
100		

Problem 1: (5 points) Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2+3y^2}{3x^2+y^2}$$

does not exist.

Problem 2: (8 points) Use polar coordinates to find the volume of the solid bounded by the paraboloids $z = 6 - x^2 - y^2$ and $z = 2x^2 + 2y^2$.

Problem 3: (12 points)

a Show that $u = sin(x - at) + \ln(x + at)$ is a solution of the wave equation $u_{tt} = a^2 u_{xx}$.

b Find the partial derivative $\frac{\partial z}{\partial s}$ when $z = x^3 - xy^2$ and where $x = 2st^2$ and $y = 3s + 4t$.

Problem 4: (6 points) Set up *but do not evaluate* the iterated integral needed to calculate the *area* of the surface obtained by considering the part of the paraboloid $z = 4 - 2x^2 - 2y^2$ that lies above the plane $z = 2$. You are expected to simplify.

<u>Problem 5:</u> (15 points) Consider the function $z = f(x, y) = x^2 - 4y^2$.

a Sketch the level curve $z = 4$.

b Use Lagrange multipliers to find the the absolute maximum z_{max} of f on the line $2x + y = 15$.

c Is there any relationship between $2x+y = 15$ and the level curves $z = z_{max}$ at their intersection? Explain.

Problem 6: (13 points)

a Explain why the function $f(x,y) = \frac{1+y}{1+x}$ is differentiable at $(1,3)$?

b Find the equation of the tangent plane of $z = f(x,y)$ at $(1,3)$.

c Find the linearization $L(x,y)$ of the function $f(x,y)$ at $(1,3)$.

Problem 7: (15 points) Consider the function $f(x, y, z) = 3x^2 - 5xy + xyz$.

a Find ∇f.

b Find the directional derivative of f at $(1, 0, 2)$ in the direction $\langle 3, -4, 0 \rangle$.

c Find the maximum rate of change of f at $(1, 0, 2)$ and the direction in which it occurs.

<u>Problem 8:</u> (12 points) Consider the function $f(x, y) = x^3 - 12xy + 8y^3$

a Find and classify all critical points of $f(x, y)$.

b Find the absolute minimum and maximum values of $f(x, y)$ in the rectangular region R defined by $0 \leq x \leq \frac{1}{2}$ and $0 \leq y \leq 1$.

Problem 9: (14 points)

a Evaluate the iterated integral $\int_1^4 \int_1^2 x + xydxdy$.

b Evaluate the double integral $\iint_D xy^2\, dA$ when D is the region bounded by $y = 1$, $x = y^2$ and $x = 0$.