MATH 253UX1 - Midterm # 3

Mahmud Alam - Fall 2020

Wednesday, November 18th 2020

Print Your First & Last Name CLEARLY	Date

INSTRUCTIONS:

- You must work on your own. You may not get help from any other person.
- This exam is **7 pages** long, not including this cover sheet.
- Once you access this exam in **gradescope** you will have **2 hours** to submit your solutions.
- Download the test and print it out. Write your solutions on the test paper. When you are done, scan the pages and save the document as a **PDF** file. Upload the file back into **gradescope**.
- If you do not have a printer you may write your answers on blank paper. But use a total 8 pages leaving the first page blank as cover sheet and please **write only one problem on each page**. Finally, make sure the problems are clearly labeled and in order. Do not submit any additional pages.
- In order to receive full credit you must show your work. Include your computations on the exam paper.
- Email me immediately if you have any trouble submitting your completed exam. You can also email me if you want to check that your exam uploaded correctly.

Total Possible Points	Score	Percent
100		

Problem 1: (12 points) Find the mass of a solid Q with density $\rho = 1$ which is bounded by the plane $x + y + z = 2$ in the first octant; find the moments with respect to each of the coordinate planes, and the center of mass; find the moments of inertia with respect to each of the coordinate axes.

Problem 2: (15 points)

a) (10 points) Use cylindrical co-ordinates to evaluate $\iiint_E y^2 dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$ and between the plane $z = 0$ and the cone $z^2 = 4x^2 + 4y^2$.

b) (5 points) Rewrite the integral integral $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2 + y^2} dz dy dx$ in cylindrical co-ordinates.

Problem 3: (15 points)

a) (10 points) Use spherical coordinate to evaluate $\iiint_E x^2 dV$, where E is the solid hemisphere $x^2 + y^2 + z^2 \leq 9, x \geq 0$.

b) (5 points) Rewrite the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xydzdydx$ in spherical co-ordinates.

Problem 4: (14 points)

a) (9 points) Use a change of variables to evaluate $\iiint_Q xz dV$ over a rectangular box where Q is the parallelopiped bounded by $y = x$, $y = x + 2$, $z = x$, $z = x + 3$, $z = 0$ and $z = 4$.

b) (5 points) Rewrite the integral $\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x, y, z) dz dy dx$ as an iterated integral in the order $dxdydz$.

Problem 5: (14 points)

a) (8 points) A thin wire has the shape of the first quadrant part of the circle with center the origin and radius a. If the density function is $\rho(x, y) = kxy$, find the mass and the center of mass of the wire.

b) (6 points) Compute the line integral $\int_C xy^2 ds$ where C is the closed curve along $x = 0$ for $-1 \leq y \leq 1$ in the downward direction then the right half circle of radius 1.

Problem 6: (15 points)

a) (7 points) Use Green's theorem to evaluate

$$\oint_C (\sin x^2 + 3y)dx + (lny + 4x)dy$$

where C is the closed curve composed of the graph of $y = \frac{x^2}{4} + 1$ for $0 \leq x \leq 2$ followed by the line segment going from $(2, 2)$ to $(0, 1)$.

b) (8 points) Evaluate $\int_c \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = (x + y^2)\mathbf{i} + xz\mathbf{j} + (y + z)\mathbf{k}$ and $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + -2t\mathbf{k}$, $(0 \leq t \leq 2)$.

Problem 7: (15 points)

a) (8 point) A force field is given by $\mathbf{F}(x, y) = \langle x^2, xy \rangle$, which is not conservative. An object moves along a straight path from the point $(1, 1)$ to $(3, 2)$. Compute the work $\mathbf{F}$ did on the object.

b) (7 points) The field
$$\mathbf{F}(x, y) = \langle e^x \sin y + y^2 + 1, e^x \cos y + 2xy + y^2 \rangle$$
is conservative. Find all potential functions for it.