Math 253 Calculus III (S. Avdonin) Name: _____ November 2, 2020

Midterm # 2

Please (circle) answers.

[100 points total]

1. [10 points] Given a surface $S = \{(x, y, z)|xy^2z^3 = 8\}$ and a point $P = (2, 2, 1)$,

(a) [5 points] Find the equation of the tangent plane to S at P.

(b) [5 points] Find the equation of the normal line to S at P.

2. [10 points] Suppose $f(x, y) = xe^y + \cos xy$.

(a) [5 points] Find the gradient of f.

(b) [5 points] Find the directional derivative of f at the point $(2, 0)$ in the direction of the vector $\mathbf{v} = \langle 3, -4\rangle$.

3. [4 points] Show that

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

does not exist.

4. [10 points] Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$:

$$yz + x \ln y = z^2$$

5. [8 points] Find and classify all the critical points of

$$z = 4 + x^3 + y^3 - 3xy.$$

6. [8 points] Use Lagrange Multiplier to find maximum and minimum values of the function $f(x, y) = x^2 - y^2$ on the circle $x^2 + y^2 = 1$.

7. [20 points] A lamina is bounded by $y = x + 2$ and $y = x^2$. Its density function is $\rho(x, y) = kx^2$.

(a) [5 points] Set up the integral to find the mass of the lamina (Do not compute).

(b) [10 points] Set up the integral to find the centroid of the lamina (Do not compute).

(c) [5 points] Find the moment of inertia of the lamina about the y axis.

8. [10 points] Rewrite the integral

$$\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x, y, z)\, dz\, dy\, dx$$

as an iterated integral in three other orders:

$$\int_?^? \int_?^? \int_?^? f(x,y,z)\, dz\, dx\, dy$$
$$\int_?^? \int_?^? \int_?^? f(x,y,z)\, dx\, dy\, dz$$
$$\int_?^? \int_?^? \int_?^? f(x,y,z)\, dx\, dz\, dy$$

Hint: You may want to sketch the region first.

9. [10 points] Use spherical coordinates to evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} y^2 \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$

10. [10 points] True or False:

(a) If $f(x,y) = y$, then $\nabla f(x,y) = 1$.

(b) The integral $\iiint_E kr^3\, dz\, dr\, d\theta$ represents the moment of inertia about the $z-$ axis of a solid E with constant density k.

(c) If $f(x,y)$ has two local maxima, then $f(x,y)$ must have a local minimum.

(d) $\int_{-1}^{2} \int_0^6 x^2 \sin(x-y)\, dx\, dy = \int_0^6 \int_{-1}^{2} x^2 \sin(x-y)\, dy\, dx.$

(e) If f is continous on $[0,1]$ then $\int_0^1 \int_0^1 f(x)f(y)\, dy\, dx = \left[\int_0^1 f(x)\, dx\right]^2$

11. Extra Credit [5 points] Evaluate

$$\iint_R \frac{x-y}{x+y}\, dA$$

where R is the square with vertices $(0,2)$, $(1,1)$, $(2,2)$, and $(1,3)$.