Math 253 Calculus III Fall 2020 (Avdonin) Name: ________________________________

Final Exam

Please circle answers.

150 points total

1. [10 points]

(a) Find a vector perpendicular to the plane through the points $A(1,0,0)$, $B(2,0,-1)$, and $C(1,4,3)$.

(b) Find the area of triangle ABC.

2. [10 points]

(a) Find a parametric equation for the line through $(4,-1,2)$ and $(1,0,0)$. **(b)** Find an equation for the plane through $(2,3,0)$ and parallel to $x+4y-5z=1$.

3. [10 points] Find the lengh of the curve $\mathbf{r}(t) = 2t^{\frac{3}{2}}\,\hat{\boldsymbol{\imath}} + \cos 2t\,\hat{\boldsymbol{\jmath}} + \sin 2t\,\hat{\boldsymbol{k}}$, where $0 \le t \le 1$.

4. [15 points]
Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

5. [10 points] If $u = x^2 y + z^3$, where $x = p + 2p^2$, $y = pe^p$, and $z = p\sin p$, find $\dfrac{du}{dp}$ when $p = 1$.

6. [5 points] Find $\dfrac{\partial z}{\partial x}$:

$$xy + y\ln z = z^2$$

7. [10 points] Find the maximum rate of change of $f(x,y) = \sin(xy)$ at point $(1,0)$ and the direction it occurs.

8. [10 points] Evaluate a triple integral to compute the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$.

9. [10 points] Change the order of integration and compute:

$$\int_0^1 \int_x^1 \cos(y^2)\,dy\,dx$$

10. [25 points] Let $\mathbf{F}(x,y,z) = 2xyz\,\hat{\boldsymbol{\imath}} + x^2z\,\hat{\boldsymbol{\jmath}} + x^2y\,\hat{\boldsymbol{k}}$

(a) [5 points] Show $\mathbf{F}$ is conservative.

(b) [10 points] Find a function f such that $\bigtriangledown f = \mathbf{F}$.

(c) [10 points] Compute $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the line segment from $(0,1,0)$ to $(2,3,4)$.

11. [5 points] Let $\mathbf{G} = \langle x^2y, xyz, -x^2y^2\rangle$. Is there a vector field $\mathbf{F} \in \mathbb{R}^3$ such that $\operatorname{Curl}\mathbf{F} = G$? Explain.

12. [10 points] Use Green's Theorem to evaluate the integral

$$\int_C y\,dx - x\,dy$$

where C consists of the line segments from $(0,1)$ to $(0,0)$, and from $(0,0)$ to $(1,0)$, and the parabola $y = 1 - x^2$ from $(1,0)$ to $(0,1)$.

13. [10 points] Let $\mathbf{F}(x,y,z) = y\,\hat{\boldsymbol{\imath}} + z\,\hat{\boldsymbol{\jmath}} + x\,\hat{\boldsymbol{k}}$ and S be the hemisphere $x^2+y^2+z^2 = 1$, $z \geq 0$, oriented upward. Evaluate the surface integral

$$\iint_S \operatorname{Curl}\mathbf{F}\cdot d\mathbf{S}.$$

14. [10 points] Use divergence theorem to compute the flux of $\mathbf{F}$ across S, where $\mathbf{F}(x,y,z) = (x^3+y^3)\,\hat{\boldsymbol{\imath}} + (y^3+z^3)\,\hat{\boldsymbol{\jmath}} + (z^3+x^3)\,\hat{\boldsymbol{k}}$, and S is the sphere described by $x^2+y^2+z^2 = 1$.

Extra Credit. [5 points] State and prove the Fundamental Theorem for Line Integrals.