Midterm # 1, explanations and corrections

Please (circle) your answers. No calculator.

[100 points total]

1. Given the points $A(1,0,0)$, $B(2,3,1)$, $C(0,2,3)$, and $D(1,1,3)$,

(a) [5 points] Find the area of the parallelogram with adjacent edges AB and AC.

(b) [5 points] Find the volume of the parallelepiped with adjacent edges AB, AC, and AD.

1. (Continued) Given the points $A(1,0,0)$, $B(2,3,1)$, $C(0,2,3)$, and $D(1,1,3)$,

(c) [5 points] Give an equation for the plane through points A, B, and C.

(d) [5 points] Find a **vector equation** for the line segment from A to D.

(e) [5 points] Find the distance between D and the plane through points A, B, and C.

2. [5 points] A constant force $\mathbf{F} = 4\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ moves an object along the line segment from $(1, 1, 0)$ to $(4, 3, 8)$. Find the work done if the distance is measured in meters and the force in newtons.

3. [5 points] Find a unit vector that's perpendicular to both $\mathbf{a} = \langle 1, -4, 3 \rangle$ and $\mathbf{b} = \langle 1, 4, -3 \rangle$.

4.

(a) [5 points] Find the center and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 + 4x - 8y + 6z - 8 = 0$$

(b) [5 points] Find an equation for the curve in which the sphere intersects the xz plane.

5. Let $\mathbf{r}(t) = \langle 4t, 3\sin(t), 3\cos(t)\rangle$

(a) [10 points] Find the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$.

(b) [5 points] Find an equation for the normal plane at $(0, 0, 3)$.

5. (Continued) Let $\mathbf{r}(t) = \langle 4t, 3\sin(t), 3\cos(t)\rangle$

(c) [5 points] Find the curvature at $t = \pi/2$.

(e) [5 points] Find the length of the curve from $t = 0$ to $t = \pi$.

6. [5 points] The velocity vector of a space ship is $\mathbf{v}(t) = \mathbf{r}'(t) = \langle t^2, 0, \cos t \rangle$ for all $t \geq 0$. Find the tangential component a_T and the normal component a_N of the acceleration.

7. A particle starts at origin and have initial velocity $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$. Its acceleration is

$$\mathbf{a}(t) = 2t\mathbf{i} + e^t\mathbf{j} - \frac{1}{(t+1)^2}\mathbf{k}$$

(a) [5 points] Find its velocity vector $\mathbf{v}(t)$.

(b) [5 points] Find its position vector $\mathbf{r}(t)$.

8. True or False

(a) [2 points] For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\quad \mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$.

(b) [2 points] For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\quad \mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

(c) [2 points] The vector $\langle 1, -1, 3 \rangle$ is parallel to the plane $x - y + 3z = 4$.

(d) [2 points] If $\mathbf{r}$ is a differentiable vector function, then $\dfrac{d}{dt}|\mathbf{r}(t)| = |\mathbf{r}'(t)|$.

(e) [2 points] If $\kappa(t) = 0$ for all t, the curve is a straight line.

9. [5 points] Find all the second partial derivatives of the function $u(x, t) = \cos(x^2 - t^2)$.

Extra Credit. [5 points] Given a smooth curve $\mathbf{r}(t)$, show that $\mathbf{T}'(t)$ is orthogonal to $\mathbf{T}(t)$.