Math 253 Exam 1 Name_____
Sept. 28, 2018

1. (5 pts. ea.) For $\vec{\mathbf{a}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, find

(a) the dot product $\vec{\mathbf{a}} \bullet \vec{\mathbf{b}}$;

(b) the cosine of the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$;

(c) the direction cosines for $\vec{\mathbf{a}}$;

$\cos\alpha =$

$\cos\beta =$

$\cos\gamma =$

(d) the component of $\vec{\mathbf{b}}$ in the direction of $\vec{\mathbf{a}}$ ($\text{comp}_{\vec{\mathbf{a}}}\vec{\mathbf{b}}$);

(e) the vector projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$ ($\text{proj}_{\vec{\mathbf{a}}}\vec{\mathbf{b}}$);

(f) the cross-product $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$.

2. (12pts.) Find the distance between the skew lines $x = 3 + 2t$, $y = 1 + t$, $z = 5 - 3t$, and $x = 2 - s$, $y = 2s$, $z = 3$.

3. (3 ea.) Give an example of an equation whose graph is
(a) a hyperboloid of two sheets;

(b) a cone;

(c) a cylinder (of any kind);

(d) a hyperbolic paraboloid.

4. (12 pts.) Find an equation for the plane through the points $(4,1,2)$, $(6,0,-1)$, and $(2,3,0)$.

5. (14 pts. total) If $\vec{\mathbf{r}}(t) = \left\langle 2t, t^2, \frac{1}{3}t^3 \right\rangle$,

(a) find the length of the curve as *t* runs from 2 to 5;

(b) find $\hat{\mathbf{T}}$, the unit tangent vector.

6. (10 pts.) Find parametric equations for the tangent line to the curve parameterized by $x = 4\sqrt{t}$, $y = e^{t^2 - t}$, $z = 2t^2 + 1$; at the point $(4,1,3)$.

7. (10 pts.) If $\vec{\mathbf{r}}(t) = \left\langle 3t - 1, -\frac{1}{3}t^3, 2t + 5 \right\rangle$, find the curvature κ of its path for $t = -1$.