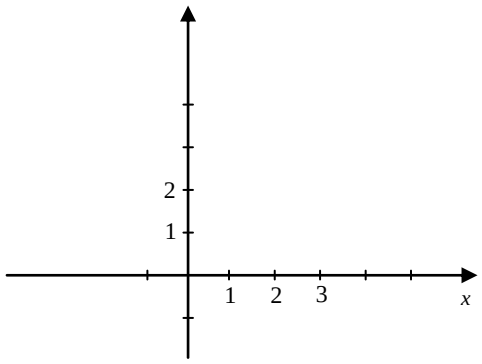
Math 253 Exam 2 Name_____
Oct. 26, 2018

1. (12 pts. total) Find the velocity, acceleration, and speed of a particle with the position function $\vec{\mathbf{r}}(t) = \langle 5\sin t, 3\cos(2t), 4t \rangle$ at $t = 0$.

2. (10 pts.) Find and sketch the domain of $f(x,y) = \dfrac{\ln\left(y+\sqrt{x}\right)}{\sqrt{x-y}}$.



3. (6 pts. ea.) Find the limit and justify your answer, or show that the limit does not exist.

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{-x^2}{\sqrt{x^4+y^4}}$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{2y^2}{5\sqrt{x^2+y^2}}$

4. (10 pts.) Use implicit differentiation to find $\dfrac{\partial z}{\partial y}$: $y^2 - 2x^3 + z^4 - e^{xyz} = 6$.

5. (10 pts.) Find an equation of the tangent plane to $z = f(x,y) = x^2 + 3xy - 2y^3$ at the point $(2,1,10)$.

6. (10 pts.) Use the chain rule to find $\frac{\partial w}{\partial t}$:

$w = x^3 + xz - 2y^3, \quad x = s\ln\left(1+rt^2\right), \quad y = rst, \quad z = t\cos(rs).$

7. (18 pts. total) For $f(xyz) = xy^2z$, find

(a) the gradient of f at the point $(3,1,-2)$;

(b) the directional derivative of f at the point $(3,1,-2)$ in the direction toward the point $(4,0,1)$;

(c) the maximum rate of change of f at the point $(3,1,-2)$ and the direction in which it occurs.

8. (18 pts. total) (a) Find the critical points for $f(x,y) = x^2 + 2y^3 + x^2y + 5y^2 + 3$.

(b) Use the Second Derivative Test to classify each critical point from part (a) as a local maximum, local minimum, or saddle point.