Math 253 Final Exam Name________________________

Dec. 10, 2018

1. (6 pts.) Find parametric equations for the line of intersection of the planes $x+y+z=6$ and $2x-y+z=4$.

2. (6 pts.) Find the distance between the point $(1,2,3)$ and the plane $4x-3y+2z=7$.

3. (6 pts.) If $\vec{\mathbf{r}}(t)=3\sin t\,\hat{\mathbf{i}}+2t^{3/2}\,\hat{\mathbf{j}}-3\cos t\,\hat{\mathbf{k}}$, find the length of the curve as t runs from 0 to 3.

4. (8 pts. total) If $\vec{\mathbf{r}}(t)=\left\langle \frac{t^3}{3}, -t^2, 3t \right\rangle$, find

(a) $\hat{\mathbf{T}}$, the unit tangent vector at the point $\left(\frac{1}{3}, -1, 3\right)$;

(b) the curvature κ at the point $\left(\frac{1}{3}, -1, 3\right)$.

5. (8 pts.) Use implicit differentiation to find $\frac{\partial z}{\partial y}$: $\tan(xy) - e^{xyz} + 3\ln x - z^3 = 4$.

6. (8 pts.) Find an equation of the tangent plane to $z = f(x,y) = 2\sqrt{x} - 5xy^2 + x - 4y$ at the point $(1,-1,2)$.

7. (8 pts. total) For $f(x,y,z) = x\sin y + 3xz^2$, find

(a) the gradient of f at the point $(2,0,1)$;

(b) the directional derivative of f at the point $(2,0,1)$ in the direction toward the point $(1,4,3)$.

8. (10 pts. total) Find the critical points for $f(x, y) = x^4 - 2x^2 + y^3 - 12y$.

(b) Use the Second Derivatives Test to classify each critical point from part (a) as a local maximum, local minimum, or saddle point.

9. (6 pts.) Find the volume of the solid that lies under the surface $z = y^3 + 4xy$, and between $y = x$ and $y = \sqrt{x}$ on the *x*-*y* plane.

10. (8 pts.) Use Lagrange multipliers to find the maximum and minimum values for $f(x,y,z)=e^{xyz}$ subject to the constraint $x^2+2y^2+z^2=24$.

11. (6 pts.) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{x^2+y^2}} \sqrt{x^2+y^2}\, dz\, dy\, dx$ by switching to cylindrical coordinates.

12. (6 pts.) Evaluate the line integral $\int_C (y\ln y)\,dx+(x\ln y+x)\,dy$ where C is the follows the curve $y=x^2+1$ from $(1,2)$ to $(2,5)$.

13. (8 pts. total) Evaluate the line integral $\oint_C xy\,dx + (2x+y)\,dy$ where C is the path from $(0,0)$ to $(1,1)$ along the curve $y = x^2$, then back to $(0,0)$ along a straight line

(a) directly;

(b) using Green's Theorem.

14. (6 pts.) For the parametric surface $\vec{\mathbf{r}}(u,v) = \langle 2v^2, u^2, 2uv \rangle$, find the area of the surface expressed over the region $0 \le u \le 3$, $0 \le v \le 2$.