Your Name

Signature (you agree to complete honestly)

Student ID #

Start Time

End Time

Page	Total Points	Score
2	20	
3	14	
4	14	
5	16	
6	17	
7	19	
Total	100	

- You will have 90 minutes to complete the exam.
- This test is closed book and you may not use a calculator.
- You may use one side of a single piece of paper (8 1/2 in. x 11 in.) of handwritten notes.
- In order to receive full credit (or partial credit in the case of incorrect solutions), you must **show your work.** Please write out your computations on the exam paper.
- Simplify all obvious expressions.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.

1. (12 points) Let $a_n = \ln\left(\dfrac{2n}{4n+5}\right)$ for $n \geq 1$.

   (a) List the first two terms in the sequence $\{a_n\}$.

   (b) Determine whether the sequence $\{a_n\}$ converges.

   (c) Let $S = \sum_{n=1}^{\infty} a_n$. Give expressions for s_1, and s_2, the first two terms of the sequence of partial sums. You do not need to simplify s_1 and s_2.

   (d) Does the series $S = \sum_{n=1}^{\infty} a_n$ converge or diverge? Explain!

2. (8 points) Give an example of a series that is (a) absolutely convergent and (b) conditionally convergent and carefully explain why in each case. If you cannot come up with an example, state the definitions for partial credit.

   (a) absolutely convergent:

   (b) conditionally convergent:

3. (14 points) Determine if the series below converge or diverge. Full credit will only be given for answers that include (1 pt) the name of the test being applied, (5 pts) a complete application of the test, including evidence that the conditions have been met, and (1 pt) a clear conclusion with justification.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

(b) $\displaystyle\sum_{n=3}^{\infty} (-1)^n \sin\left(\frac{2\pi}{n}\right)$

4. (14 points) Determine if the series below converge or diverge. Full credit will only be given for answers that include (1 pt) the name of the test being applied, (5 pts) a complete application of the test, including evidence that the conditions have been met, and (1 pt) a clear conclusion with justification.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 5n}{2n + 4n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2 + \sin(3n)}{1 + 3^n}$

5. (16 points) Find the radius and interval of convergence of the following series. If applicable, clearly explain why the series does or does not converge at the endpoints.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(3x)^n}{(n+1)^n}$

b) $\displaystyle\sum_{n=0}^{\infty} (n+1)!(x-2)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{\sqrt{n} \cdot 4^n}$

6. (13 points) Find the sum of the following series exactly – simplify answers. If the series diverges explain why.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{3}{2^n n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\pi^{2n}}{4^{2n}(2n)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot 3^{n+2}}{4^n}$

7. (4 points) Determine the number of terms in the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{4}{n^2}$ that we need to ensure that the $|\text{error}| \leq 0.01$.

8. (10 points) Given $f(x) = \dfrac{x^4}{x^3+8}$.

   (a) Find and simplify a power series for $f(x)$ using a geometric series and give the radius of convergence.

   (b) Using your result from (a), evaluate $\int f(x)\,dx$ as a power series. State the radius of convergence.

9. (6 points) Find the Taylor Series for $f(x) = 3/x$ centered at $a = 2$ using the definition. You must write your answer using summation notation.

10. (3 points) Find and simplify a Maclaurin series for $f(x) = x^2 \sin(5x^3)$ using a known Maclaurin series.