Your Name

Your Signature

Start Time

End Time

Problem	Total Points	Score
1	12	
2	10	
3	13	
4	6	
5	10	
6	11	
7	12	
8	14	
9	12	
Extra Credit	(10)	
Total	100	
Percent	100 %	

- This test is closed note and closed book, with the exception of a single side of a single 3×5 inch handwritten note card. You must submit your note card with your exam.
- You are not allowed to use a calculator. You are not allowed to share notes.
- In order to receive full credit, you must **show your work** and **justify your answers.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- Where *appropriate*, **PLACE A BOX AROUND YOUR FINAL ANSWER to each question**. (Don't bother where it doesn't add clarity to your work.)
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

$\boxed{1}$ Determine whether the series below converge or diverge by *using a suitable comparison test.* *Justify* your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{2n^{3/2}+6}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n-2n}$

$\boxed{2}$ Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+2n}$.

(a) Show that the series is convergent. *Justify* your answer.

(b) How many terms of the series do we need to add in order to find the sum to within an accuracy of $|\text{error}| < 0.0005$? *Justify* your answer.

$\boxed{3}$ Suppose $a_k > 0$ for all k, and $\sum_{n=1}^{\infty} a_k$ converges. For each problem below, *either* **justify** *why the statement is true*, or **provide a counterexample** *that demonstrates the claim is false.*

To avoid any possible confusion: $\sum_{n=1}^{\infty} 2^n$ is a counterexample to the claim "Every geometric series is convergent."

(a) $\sum_{n=1}^{\infty} 5a_k$ must always converge.

(b) $\sum_{n=1}^{\infty} 1/a_k$ must always converge.

(c) $\sum_{n=1}^{\infty} (a_k)^2$ must always converge.

(d) $\sum_{n=1}^{\infty} \sqrt{a_k}$ must always converge.

$\boxed{4}$ Determine whether the series below converge or diverge by *using a suitable test*. *Justify* your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2 - n\cos n + 382}{(3n+5)^2}$

(b) $\displaystyle\sum_{n=17}^{\infty} \frac{1}{\sqrt[4]{n}+4}$

(c) $\displaystyle\sum_{n=17}^{\infty} \frac{1}{\sqrt[4]{n}-4}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{e^{n^2}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos((n+4)\pi)}{\sqrt[5]{n^6+n}}$

(f) $\displaystyle\sum_{n=1}^{\infty} a_n$, where $a_1 = 3$ and $a_{n+1} = \frac{\cos(a_n)a_n}{3n-5}$

(g) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{\arctan(x)}{n+5}\right)^{2n}$

$\boxed{5}$ Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x-2)^{n+1}}{3^n+n^2}.$$

$\boxed{6}$ Express the function $f(x) = x^3 \ln |x - 3|$ as a power series centered at $x = 0$ and determine the radius of convergence. (Hint: You should *not* use Taylor's method for this problem.)

$\boxed{7}$ What are the first three terms of the Taylor series of $g(x) = \cos(2x)e^{3x}$? Simplify your answer.

$\boxed{8}$

(a) Find the Taylor polynomial of degree 3 ($T_3(x)$) for $f(x) = \sqrt{x}$ centered at $x = 9$.

(b) Use $T_3(x)$ to estimate $\sqrt{10}$. According to Taylor's inequality, at worst, how far off will your estimate be from the true value?

$\boxed{9}$ (Extra Credit points)

(a) Use series to estimate $\sin(2)$ correct to within 0.0001. To get full credit your solution should:

- Make a good choice for where to center the series you use.
- Include all supporting computations.
- Be possible to calculate *without* using any non-numeric keys on a calculator other than $+, -, \times, \div$ and $=$.