

Your Name

Your Signature

Start Time

End Time

Problem	Total Points	Score
1	16	
2	13	
3	10	
4	20	
5	25	
6	16	
Extra Credit	(10)	
Total	100	
Percent	100 %	

- This test is closed note and closed book.
- You are not allowed to use a calculator. You are not allowed to share notes.
- In order to receive full credit, you must **show your work** and **justify your answers**. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** YOUR FINAL ANSWER **to each question** *where appropriate*. (Don't bother where it doesn't add clarity to your work.)
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

- 1 Sketch the region enclosed by the curves $y = x^3 + x + 1$ and $y = 5x + 1$ and find its area.

- 2 For each integral below, indicate a technique that can be used to evaluate the integral, and then apply the technique to rewrite the integral as a simpler one. Finally, without actually solving the integral, indicate how to proceed from there. (Note: Looking up a form in a table of integrals is *not* a valid response, and some problems may involve more than one technique.)

(a) $\int \frac{dx}{\sqrt{16 - x^2}}$

(b) $\int_1^2 \frac{x - 7}{\sqrt{16 - x^2}} dx$

(c) $\int t^3 e^{-t^2} dt$

(d) $\int \frac{3x+2}{x^2+6x+8} dx$

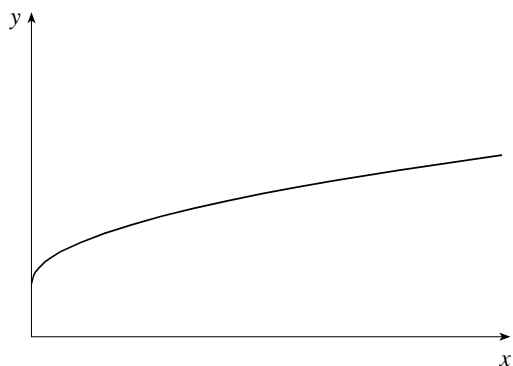
(e) $\int t^3 (2+t^2)^{5/2} dx$

3 Consider the integral $\int_2^3 \frac{dx}{(x-2)^{3/2}}$.

(a) Why is this an improper integral?

(b) Determine if the integral converges or diverges. If it converges, compute its value.

- 4 Let $f(x)$ be the function graphed below.



Four students approximated the area under $f(x)$ from 0 to 1. They all used the same number of subintervals, but they each used a different method. Here are their results:

Gary	2.453
Vara	2.638
Tariq	2.555
Patrick	2.178

Which student used which method? Explain.

Left endpoint approximation:

Right endpoint approximation:

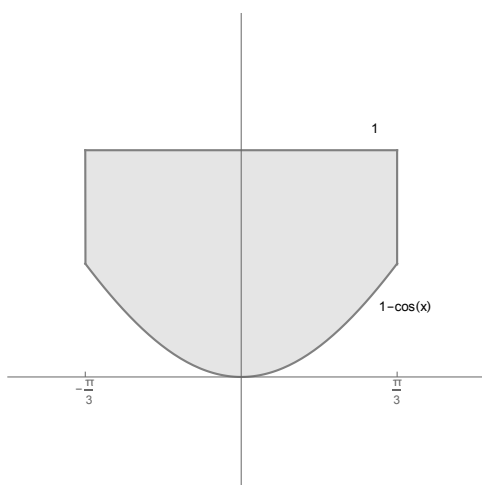
Midpoint rule:

Trapezoid rule:

- 5 How large should n be to guarantee that the Trapezoid rule estimation is accurate to within 0.00001 for the integral $\int_0^2 \sqrt{3x+3} \, dx$?

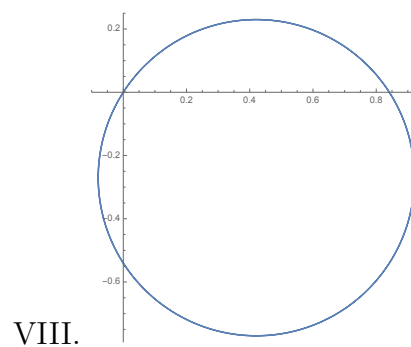
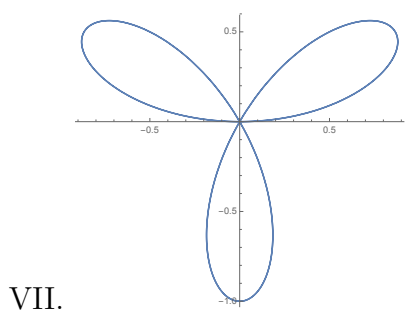
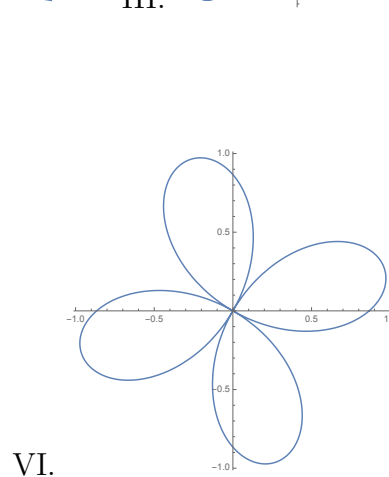
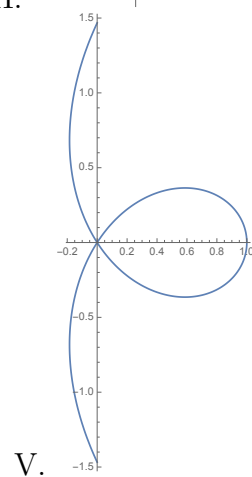
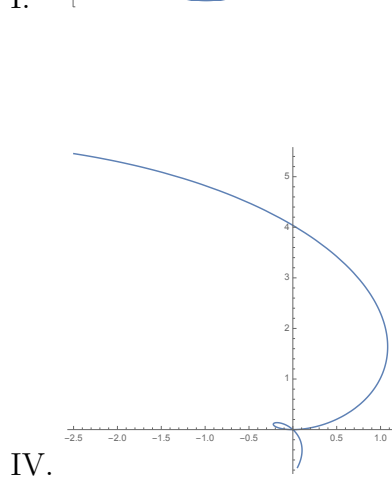
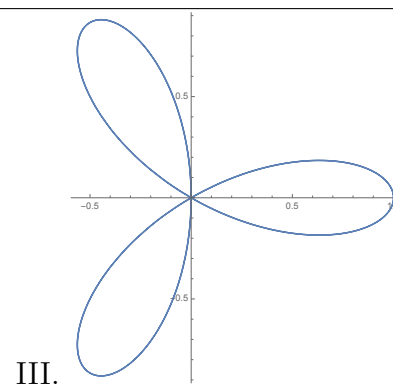
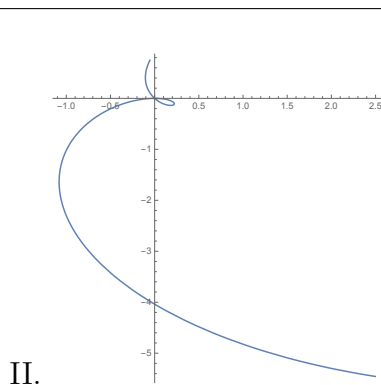
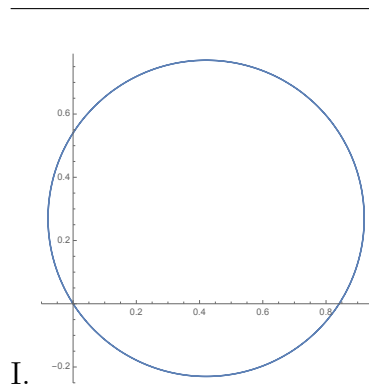
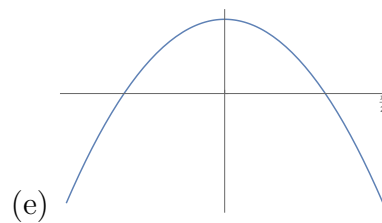
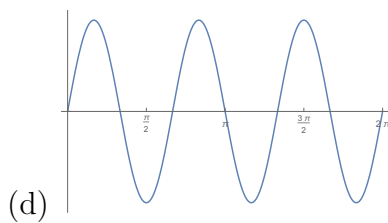
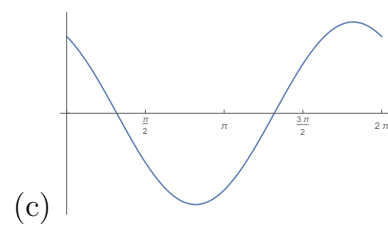
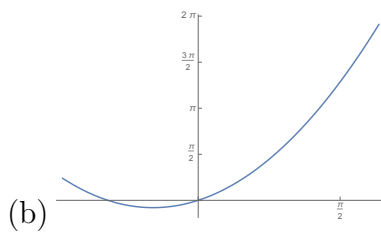
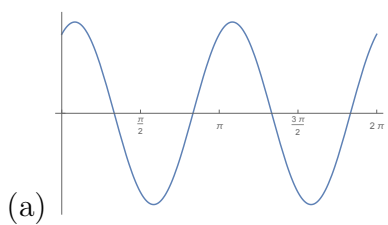
- 6 Find the area of the surface formed by revolving the curve $y = x^2$, $1 \leq x \leq 3$ about the y -axis.

- 7 Find the centroid of the region graphed below.



- 8 The graphs on the top half of the page show how r varies with θ in rectangular/Cartesian form. The graphs on the bottom half of the page shows the polar graphs of these functions. Match up each of the graphs in the top half with one of the graphs in the bottom half so that each pair corresponds to one equation for r as a function of θ .

You do not have to justify your answers.



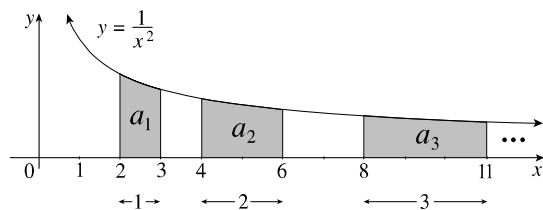
- 9 Determine whether the sequence is convergent or divergent. If it is convergent, find its limit. Justify your answers.

(a) $\frac{1}{5}, -\frac{3}{10}, \frac{5}{17}, -\frac{7}{26}, \frac{9}{37}, \dots$

(b) $a_n = \frac{n^4}{n^3+n+1}$

10 Determine whether the series below converge or diverge. Justify your answers.

(a) $\sum_{n=1}^{\infty} a_n$ where a_n are illustrated below.



(b) $\sum_{n=1}^{\infty} \frac{(-16)^n}{n^{3n}5^{2n}}$

(c) $\sum_{n=1}^{\infty} \frac{(n+1)(n^2-2)}{5n^3-3n+2}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(\sqrt{2})^n}{n^6}$

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- (a) What is the Maclaurin series for $x \sin(x^2)$?
- (b) What are the first three non-zero terms of this Maclaurin series?
- (c) What is $f^{(8)}(0)$ of $f(x) = x \sin(x^2)$? Justify your answer. (You should not have to take a derivative to figure this out.)

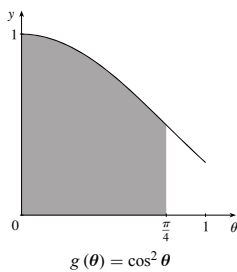
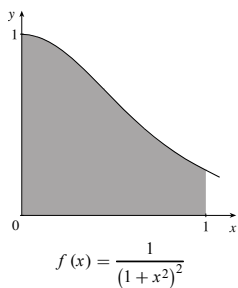
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- (a) What is the Taylor $T_2(x)$ polynomial of degree 2 for the function $f(x) = \frac{1}{1+3x}$ centered at $x = 2$?

- (b) Estimate the error in using $T_2(x)$ to approximate $f(x)$ on the interval $[1, 3]$.

13 (extra credit points)

(a) Show that the areas of the two shaded regions below are the same:



(b) Graph $r(\theta) = 1 - 2 \sin \theta$ in both cartesian and polar coordinates.

(c) Find the area enclosed by the inner loop of the curve $r(\theta) = 1 - 2 \sin \theta$.