Math 252
Exam II

Key
Name

"I can tell you that only a fool destroys useful things merely because he doesn't like them."
--Avram Davidson

1 (14 Points) Find the Maclaurin series for $(x^3+6)^{-1}$

$$\frac{1}{6}\frac{1}{1+\frac{x^3}{6}} = \frac{1}{6}\frac{1}{1-\left(-\frac{x^3}{6}\right)}$$

$$= \sum_{k=0}^{\infty} \frac{1}{6}\left(-\frac{x^3}{6}\right)^k$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{6^{k+1}} x^{3k}$$

2 (15 Points). Evaluate $\lim_{n\to\infty} n \ \mathrm{Sin}(\frac{5}{n})$.

Go to the Continuous Counterpart. (You cannot take the derivative of a Sequence.)

$$\lim_{x\to\infty} x \sin\left(\frac{5}{x}\right)$$

$$= \lim_{x\to\infty} \frac{\sin\left(\frac{5}{x}\right)}{x^{-1}}$$

$$= \lim_{x\to\infty} \frac{\cos\left(\frac{5}{x}\right)\left(-\frac{5}{x^2}\right)}{-1/x^2}$$

$$= \lim_{x\to\infty} 5\cos\left(\frac{5}{x}\right) = 5\cdot\cos(0) = 5$$

3 (26 Points). In both of the following, tell if the series converges or diverges. State the name of the test you are using and show your work.

(a) $\sum_{k=1}^{\infty} \frac{4}{9+k}$ Diverges. Limit Comparison test with the divergent harmonic series.

$$\frac{\frac{4}{9+K}}{1/K} = \frac{4K}{9+K} = \frac{4}{\frac{9}{K}+1} \to \frac{4}{0+1} = 4$$

(b) $\sum_{n=4}^{\infty} \frac{3n^2+5n}{9n^2+11n+5}$

Diverges by Divergence Test.

$$\frac{3n^2+5n}{9n^3+11n+5} = \frac{3+\frac{5}{n}}{9+\frac{11}{n}+\frac{5}{n^2}} \to \frac{3+0}{9+0+0}$$

$$= \frac{1}{3} \neq 0.$$

4 (15 Points). Find the radius of convergence for $\sum_{k=1}^{\infty} \frac{7(x-11)^k}{5^k k^3}$.

We'll use Root Test. The Ratio Test also works.

$$\sqrt[n]{\left|\frac{7(x-11)^n}{5^n n^3}\right|}$$

$$= \frac{\sqrt[n]{7}}{5} \frac{\sqrt[n]{|x-11|^n}}{(\sqrt[n]{n})^3}$$

$$= \frac{7^{\frac{1}{n}}}{5} \frac{|x-11|}{(\sqrt[n]{n})^3} \longrightarrow \frac{7^0 |x-11|}{5 \cdot 1}$$

$$= \frac{1}{5}|x-11| < 1. \qquad |x-11| < 5$$

$$R = 5$$

5 (14 Points). Express as a closed form expression: $\sum_{k=0}^{\infty} \frac{x^{2k+3}}{4^k}$.

$$x^3 \sum_{k=0}^{\infty} \frac{x^{2k}}{4^k}$$

$$= x^3 \sum_{k=0}^{\infty} \left(\frac{x^2}{4}\right)^k$$

$$= x^3 \frac{1}{1 - \frac{x^2}{4}}$$

$$= \frac{4x^3}{4 - x^2}$$

6 (16 Points). Bald answers are allowed on this problem. Give an example of a series that:

Many Examples

(a) is conditionally convergent

$$\sum \frac{(-1)^n}{n}$$

(b) is absolutely convergent

$$\sum \frac{1}{n^2}$$

(c) diverges

$$\sum \frac{1}{n}$$

(d) converges to four

$4+0+0+0+\cdots$