Your Name

Start Time

End Time

Page	Total Points	Score
2	10	
3	10	
4	11	
5	8	
6	8	
7	7	
8	10	
9	8	
10	8	
11	10	
12	10	
Total	100	

- You will have 2 hours to complete the test.
- This test is closed notes except for a 4 by 6 inch note card with hand written notes on both sides.
- No calculators allowed.
- Label any diagrams so as to indicate axes labels and scale.
- In order to receive full credit, you must **show your work.** Please write out your computations on the exam paper.
- When a problem asks you to **set up only** you do NOT need to simplify the integrand (the expression inside the integral sign) at all.
- When determining convergence or divergence of a series, state the test that is being applied. Common abbreviations are acceptable. **PLACE A BOX AROUND YOUR FINAL ANSWER to each question** where appropriate.

1. (20 points) Evaluate the following integrals.

(a) $\displaystyle\int_0^{\pi/2} \sin^3 x \cos^4 x dx$

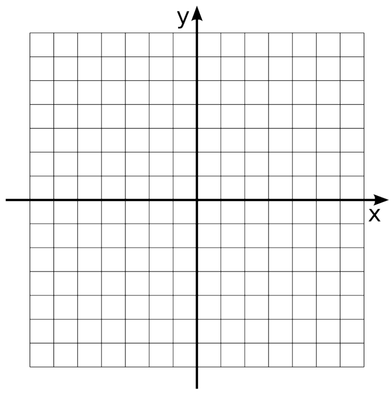
(b) $\displaystyle\int \frac{10}{(x-3)(x^2+1)} dx$

(c) $\int \sin^{-1}(5x)dx$

(d) $\int \frac{x^2}{(9-x^2)^{3/2}}dx$

2. (11 points) Let R be the region bounded by the graphs of $f(x) = 1 + x^2$, $g(x) = 3 - x^2$.

(a) Graph the region and then set up, but do not solve, an integral that gives the **area** of R.



(b) Set up, but do not solve, an integral that finds the **volume** of the solid when R is rotated about the x-axis.

(c) Set up, but do not solve, an integral that finds the volume of the solid when R is rotated about the line $x = 4$.

(d) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is twice the length of its base in the region R. Set up, but do not solve, an integral that gives the volume of this solid.

3. (4 points) Let $a_n = \cos\left(\frac{n+1}{3n^2+4}\right)$.

(a) Determine whether the sequence a_n converges. If it is convergent determine what it converges to.

(b) Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

4. (4 points) Find the sum of the following series exactly.

a) $\sum_{n=0}^{\infty} \frac{3 \cdot (-2)^n}{n!}$

b) $\sum_{n=1}^{\infty} (-5)^{n+1} 3^{-2n}$

5. (4 points) Find the Taylor series for the function $f(x) = e^{-2x}$ centered at the point $a = 4$. Give your answer in summation notation.

6. (4 points) Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$.

   (a) Compute S_3.

   (b) Determine the error of S_3 as an approximation of the sum of the given series.

7. (7 points)

(a) Determine whether the improper integral $\displaystyle\int_1^\infty \frac{x}{x^2+9}dx$ converges or diverges. Evaluate it if it is convergent. [Use proper notation for full credit.]

(b) Use the integral test, and your answer from (a), determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+9}$ converges or diverges. You must explicitly verify that the integral test applies to this series. No credit will be given if another test is used.

8. (10 points) Determine whether the following series converge or diverge. You must clearly explain your reasoning.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{2+n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+1}$

9. (8 points) Find the radius of convergence and the interval of convergence of the following series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n6^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n!}$

10. (8 points) Let $\mathcal{R}$ be the region bounded by $y = e^{2x}$ and $y = 0$, $0 \leq x \leq 2$.

(a) Sketch the region and find the area of $\mathcal{R}$.

(b) Find the centroid of the the region $\mathcal{R}$.

11. (10 points) Consider $x = e^t$, $y = te^{-t}$.

(a) Find and simplify $\dfrac{dy}{dx}$.

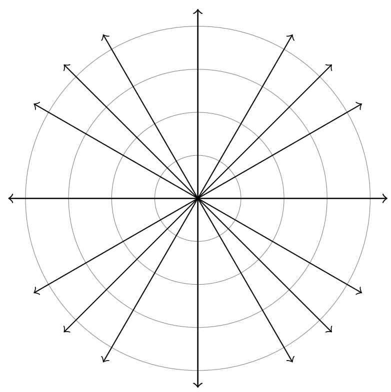
(b) Determine the location of any horizontal tangents. If none exist, explain why.

(c) Find and simplify $\dfrac{d^2y}{dx^2}$.

(d) Determine the values of t for which the curve is concave up.

12. (10 points) Consider the curve $r = 3\cos(2\theta)$.

(a) Sketch the curve $r = 3\cos(2\theta)$.



(b) Find the area enclosed by one petal.

(c) Set up, but do not solve, an integral that gives the length of the polar curve when it is traversed once.