

Your Name

Start Time

End Time

Page	Total Points	Score
2	20	
3	10	
4	14	
5	14	
6	14	
7	18	
8	10	
Total	100	

- You will have 1 hour to complete the test.
- This test is closed notes and closed book and you may not use a calculator.
- In order to receive full credit, you must **show your work**. Please write out your computations on the exam paper.
- **PLACE A BOX AROUND** **YOUR FINAL ANSWER** to each question where appropriate.

1. (12 points) Let $a_n = \sin\left(\frac{\pi n^2}{3 + 2n^2}\right)$ for $n \geq 1$.

(a) List the first two terms in the sequence $\{a_n\}$.

(b) Determine whether the sequence $\{a_n\}$ converges.

(c) Let $S = \sum_{n=1}^{\infty} a_n$. Give expressions for s_1 , and s_2 , the first two terms of the sequence of partial sums. You do not need to simplify s_1 and s_2 .

(d) Does the series $S = \sum_{n=1}^{\infty} a_n$ converge or diverge? Explain using a relevant theorem.

2. (8 points) Give an example of a series that is (a) absolutely convergent and (b) conditionally convergent and carefully explain why in each case. If you cannot come up with an example, state the definitions for partial credit.

(a) absolutely convergent

(b) conditionally convergent

3. (6 points)

(a) Find the values of x so that $\sum_{n=0}^{\infty} 2(4x)^n$ converges. Give your final answer as an interval.

(b) For an x value in that interval, what is the sum of the series?

4. (2 points) For what values of a will $\sum_{n=1}^{\infty} \frac{1}{n^a}$ converge?

5. (2 points) For what values of b will $\sum_{n=1}^{\infty} b^n$ converge?

6. (14 points) Determine if the series below converge or diverge. Full credit will only be given for answers that include (1 pt) the name of the test being applied, (6 pts) a complete application of the test, including evidence that the conditions have been met, and (1 pt) a clear conclusion with justification.

(a)
$$\sum_{n=2}^{\infty} \frac{3}{n(\ln n)^2}$$

(b)
$$\sum_{n=1}^{\infty} \frac{2 \sin(3n)}{\sqrt{4 + n^3}}$$

7. (14 points) Determine if the series below converge or diverge. Full credit will only be given for answers that include (1 pt) the name of the test being applied, (6 pts) a complete application of the test, including evidence that the conditions have been met, and (1 pt) a clear conclusion with justification.

(a)
$$\sum_{n=1}^{\infty} \frac{n^4 + 5}{n^2 + 2n^5}$$

(b)
$$\sum_{n=1}^{\infty} 3e^{1/n}$$

8. (14 points) Determine if the series below converge or diverge. Full credit will only be given for answers that include (1 pt) the name of the test being applied, (6 pts) a complete application of the test, including evidence that the conditions have been met, and (1 pt) a clear conclusion with justification.

(a)
$$\sum_{n=1}^{\infty} \left(\frac{1 - 3n^2}{2n^2 + 2} \right)^{3n}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n2^{2n}}{n!}$$

9. (18 points) Find the sum of the following series exactly. If the series diverges explain why.

(a)
$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+1}}{4 \cdot 2^n}$$

(b)
$$\sum_{n=1}^{\infty} \left(\frac{2}{\ln(n+2)} - \frac{2}{\ln(n+1)} \right)$$

(c)
$$\sum_{n=1}^{\infty} 2^{n+2} 3^{-n}$$

10. (10 points) Consider the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{e^n}$.

(a) Show the alternating series converges.

(b) Approximate the sum by using the first three terms. Simplify each term, but do not add together. What is the name of this sum?

(c) What is the error of the estimate in part (b)?