Your Name

Instructor Name

Start Time

End Time

Page	Total Points	Score
2	13	
3	16	
4	18	
5	18	
6	16	
7	19	
Total	100	

- You will have 60 minutes to complete the test.
- This test is closed notes and closed book and you may not use a calculator.
- Label any diagrams so as to indicate axes labels and scale.
- In order to receive full credit, you must **show your work.** Please write out your computations on the exam paper.
- Simplify all answers by fully distributing any constants.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.
- Clearly explain all of your solutions using the relevant theorems.

1. (13 points) Suppose we are given a function and its power series centered at 0. So $f(x) = \sum_{n=0}^{\infty} a_n x^n$ with finite, positive radius of convergence, $R > 0$.

   (a) Write out the series for $f(x)$ in "expanded form".

   (b) Find $T_2(x)$, the second degree Taylor polynomial that approximates $f(x)$.

   (c) Evaluate $f(0)$.

   (d) Find $f''(0)$.

   (e) T/F: $f(R+1) = \sum_{n=0}^{\infty} a_n (R+1)^n$. Justify.

2. (16 points) Find the radius and interval of convergence of the following series. If applicable, clearly explain why the series does or does not converge at the endpoints.

a) $\sum_{n=0}^{\infty}(2n)!(x+5)^n$

b) $\sum_{n=0}^{\infty}\frac{(x-3)^n}{n!}$

(c) $\sum_{n=1}^{\infty}\frac{(-1)^n(x+1)^n}{n^2\cdot 4^n}$

3. (18 points) Find the sum of the following series exactly. If the series diverges explain why.

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{5^n}{2n!}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\pi^{2n+1}}{6^{2n+1}(2n+1)!}$

(c) $1 + 2 + \frac{4}{2!} + \frac{8}{3!} + \frac{16}{4!}...$

4. (10 points) Given $f(x) = \dfrac{x^2}{x^4 + 16}$.

   (a) Find and simplify a power series for $f(x)$ using a geometric series and give the radius of convergence.

   (b) Using your result from (a), evaluate $\int f(x)dx$ as a power series. Simplify your result and state the radius of convergence.

5. (8 points) Find the Taylor Series for $f(x) = \dfrac{1}{x}$ centered at $a = 3$ using the definition. You must write your answer using summation notation. Simplify and cancel if applicable.

6. (8 points) Find the Taylor Series for $f(x) = e^{3x}$ centered at $a = -2$ using the definition. You must write your answer using summation notation. Simplify and cancel if applicable.

7. (8 points) Use differentiation to find a power series representation of $f(x) = \dfrac{3}{(2-x)^2}$ and state the radius of convergence. [Hint: $\displaystyle\int f(x)\,dx = \frac{3}{2-x}$.]

8. (8 points) Find and simplify a Maclaurin series for the following functions using common Maclaurin series.

   (a) $f(x) = x^5 e^{2x}$

   (b) $f(x) = \cos\left(2x^4\right)$

9. Let $f(x) = e^{-x}$.

   (a) (5 points) Find $T_3(x)$, the third degree Taylor polynomial that approximates $f(x)$.

   (b) (3 points) Approximate e^{-2} using $T_3(x)$. Simplify your expression.

   (c) (3 points) Use the alternating series remainder to estimate the accuracy of the estimate found in part (b).