Your Name

Start Time

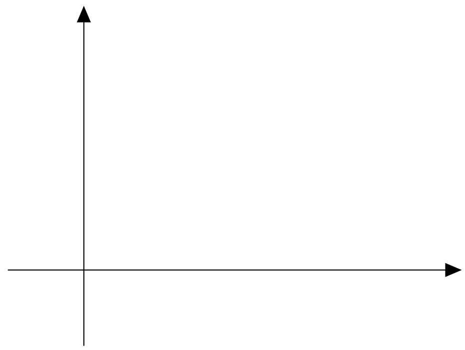
End Time

Page	Total Points	Score
2	10	
3	17	
4	22	
5	15	
6	21	
7	15	
8	(5 Extra Credit)	
Total	100	

- You will have 1 hour to complete the test.
- This test is closed notes and closed book and you may not use a calculator.
- Label any diagrams so as to indicate axes labels and scale.
- In order to receive full credit, you must **show your work.** Please write out your computations on the exam paper. Simplify answers where appropriate.
- Carefully read all problems as some problems are set-up only.
- **PLACE A BOX AROUND YOUR FINAL ANSWER to each question** where appropriate.

1. (10 points) Consider the region bounded by $y = x$ and $y = \sqrt[3]{x}$ for $0 \le x \le 8$.

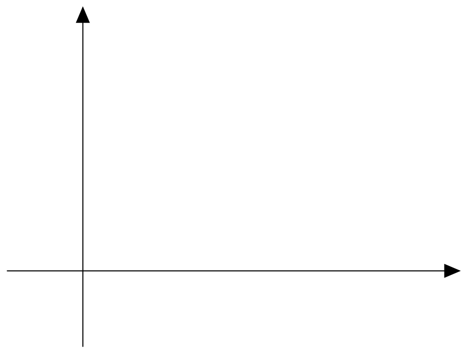
   (a) (2 points) Sketch the region bounded by the curves. Clearly label each curve and label any important points.

   (b) (8 points) Find the area of this region.

2. (10 points) In this problem you are going to find the number a such that the line $x = a$ divides the area under the curve $y = x^2$ into two regions of equal area for $1 \leq x \leq 3$.

   (a) (3 points) Sketch a rough graph of the region and clearly indicate what you are looking for.

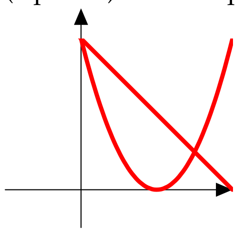
   

   (b) (7 points) Now, find the number a such that the line $x = a$ divdes the area under the curve $y = x^2$ into two regions of equal area for $1 \leq x \leq 3$. Explain why your answer is reasonable.

3. (7 points) The base of a solid is the region bounded by $y = 2 - x^2$ and the x-axis. Cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

4. (22 points) Consider the region bounded by $y = x^2 - 4x + 4$, $y = 4 - x$, which is graphed below. Set up, but do not solve, an integral that finds the volume of this region when it is rotated about each of the following axes. **You do NOT need to simplify the integrand.** State the method that you are using. A sketch of the region has been provided for you.

   (a) (2 points) Find the points of intersection of $y = x^2 - 4x + 4$ and $y = 4 - x$.



   (b) (5 points) x-axis

   (c) (5 points) y-axis

   (d) (5 points) $x = -3$

   (e) (5 points) $y = 5$

5. (15 points) The temperature in a certain city (in °F) t hours after 8 AM was modeled by the function

$$T(t) = 5\sin\left(\frac{\pi t}{12}\right) - 20.$$

(a) (8 points) Use this equation to find the average temperature during the period from 8 AM to 8 PM. Give an exact answer with proper units AND then, using $\pi \approx 3$ give a rough estimate of what the average temperature is to the nearest whole number.

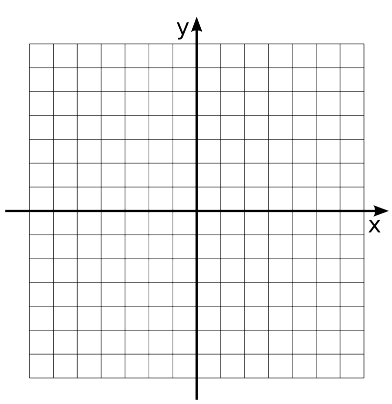
(b) (2 points) Explain why the Mean Value Theorem for integrals applies to the equation $T(t)$ on any interval $[a, b]$.

(c) (5 points) Find the time $t = c$ such that $T(c)$ is equal to the average value from part (a). Use the exact, not the approximate answer to do this. Give units with your answer.

6. (8 points) A spring has a natural length of 10 cm. If an 80-N force is required to stretch the spring to a length of 30 cm, how much work is required to stretch the spring an additional 10 cm? Give your answer with proper units.

7. (7 points) Find the exact length of the curve $y = 3 - 4x^{3/2}$ for $0 \leq x \leq 1$.

8. (6 points) Set up, simplify, but do not solve an integral describing the surface area obtained by rotating the region $y = 2 + \sqrt{x}$ between $x = 0$ and $x = 4$ about the y-axis. Begin by sketching the curve.



9. (15 points) Consider the region bounded by $y = \cos(2x)$, $x = 0$, and $y = 0$ on the interval $[0, \pi/8]$.

(a) (5 points) Sketch curves on the interval from $[0, \pi]$, shade the region, and then find the area bounded by the curves.

(b) (10 points) Find the centroid (also known as the center of mass) of this region.

10. (Extra Credit 5 points) A tank is full of water. Find the work required to pump the water out of the spout. Use 9.8 m/s^2 for the acceleration due to gravity and assume that water has a density of 1000 kg/m^3. (Note: Partial credit will be awarded for correct set-up and stating the correct units of the final answer.)