UAF eLEARNING & DISTANCE EDUCATION
University of Alaska Fairbanks

MATH F252X - Final Exam

Park - Fall 2018

Wednesday, December 12

Date:

Print Your First & Last Name CLEARLY	Proctor Name

Start Time:	End Time:

- **Student Responsibilities**
  - It is the student's responsibility to keep track of their time. Students will be penalized for using more than the allotted time. **The total allotted time on this exam is 120 minutes.**
  - It is the student's responsibility to ensure all pages are included in order with the exam. The exam is 11 pages, including the cover sheet.
- **Specific Instructions**
  - You will have 2 hours to complete the test.
  - This test is closed notes except for a 4 by 6 inch note card with hand written notes on both sides.
  - No calculators allowed.
  - Label any diagrams so as to indicate axes labels and scale.
  - In order to receive full credit, you must **show your work.** Please write out your computations on the exam paper.
  - When a problem asks you to **set up only** you do NOT need to simplify the integrand (the expression inside the integral sign) at all.
  - When determining convergence or divergence of a series, state the test that is being applied. Common abbreviations are acceptable. **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.

Total Possible Points	Score	Percent
100		

1. (20 points) Evaluate the following integrals.

   (a) $\int_0^{\pi/2} \sin^5 x dx$

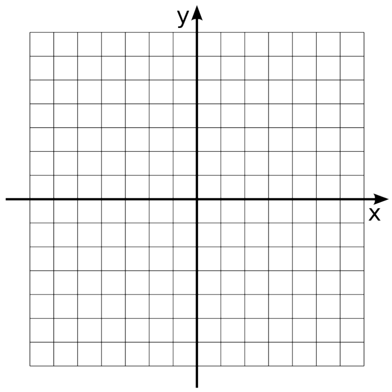
   (b) $\int \frac{2x^2 - x + 4}{x^3 + x} dx$

(c) $\int \sin^{-1}(5y)dy$

(d) $\int \frac{x^2}{(9-x^2)^{3/2}}dx$

2. (11 points) Let R be the region bounded by the functions $f(x) = 3x - x^2$ and $g(x) = x$.

(a) (2 points) Graph the region and then set up, but do not solve, an integral that gives the **area** of R.



(b) (3 points) Set up, but do not solve, an integral that finds the **volume** of the solid generated when R is rotated about the y-axis.

(c) (3 points) Set up, but do not solve, an integral that finds the volume of the solid generated when R is rotated about the line $y = -1$.

(d) (3 points) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Set up, but do not solve, an integral that gives the volume of this solid.

3. (4 points) Let $a_n = \tan^{-1}\left(\dfrac{n^2+1}{n^2+n+2}\right)$.

   (a) Determine whether the sequence a_n converges or diverges. If it is convergent determine what it converges to.

   (b) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges. Justify your answer.

4. (5 points) Find the sum of the following series exactly.

   a) $\displaystyle\sum_{n=0}^{\infty} \frac{5\cdot(-1)^n 2^n}{n!}$

   b) $\displaystyle\sum_{n=1}^{\infty} 3^{2n+1}10^{-n}$

5. (4 points) Find the Taylor series for the function $f(x) = 1/x$ centered at the point $a = -2$ using the definition. Give your answer in summation notation.

6. (8 points)

   (a) (4 points) Determine whether the improper integral $\int_2^\infty \frac{\ln x}{x} dx$ converges or diverges. Evaluate it if it is convergent.

   (b) (4 points) Use the integral test, and your answer from (a), determine whether $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ converges or diverges. You must explicitly verify that the integral test applies to this series. No credit will be given if another test is used.

7. (2 points) Determine the number of terms in the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ that we need to add to ensure $|\text{error}| \leq 0.001$.

8. (10 points) Determine whether the following series converge or diverge. You must clearly explain your reasoning and state any relavant tests by name.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$

9. (8 points) Find the radius of convergence and the interval of convergence of the following series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x+3)^n}{n^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{n \cdot 5^n}$

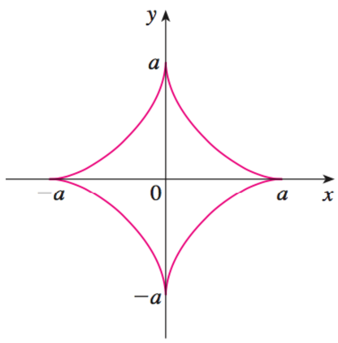
10. (10 points) Let $\mathcal{R}$ be the region bounded by $y = 3\sin(x)$ and $y = 0$, $0 \leq x \leq \pi/2$.

    (a) (2 point) Sketch the region and find the **AREA** of $\mathcal{R}$ here in PART A.

    (b) (8 points) Find the centroid of the the region $\mathcal{R}$.

11. (9 points) Consider the parametric curve where $x = a\sin^3\theta$ and $y = a\cos^3\theta$. A graph is given below in part (a).

(a) (2 points) Find and simplify $\dfrac{dy}{dx}$.



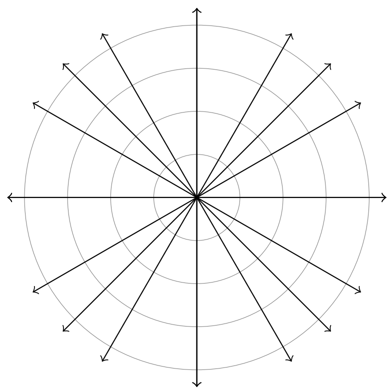
(b) (2 points) Determine the location of any vertical tangents for $0 \leq \theta \leq 2\pi$. Give the θ values only.

(c) (3 points) Set up, but do not solve, an integral that gives the area of the region.

(d) (3 points) Set up, but do not solve, an integral that gives the length of the curve.

12. (9 points) Consider the curve $r = 2\cos(3\theta)$.

(a) (3 points) Sketch the curve $r = 2\cos(3\theta)$.



(b) (4 points) Find the area enclosed by one petal.

(c) (2 points) Set up, but do not solve, an integral that gives the length of the polar curve.