MATH F252X - Midterm # 3

Thursday, November 15th

Park - Fall 2018

Date:

Print Your First & Last Name CLEARLY	Proctor Name

Start Time:	End Time:

- **Student Responsibilities**
  - It is the student's responsibility to keep track of their time. Students will be penalized for using more than the allotted time at a rate of 2 points per five minutes. **The total allotted time on this exam is 60 minutes.**
  - It is the student's respsonsibility to ensure all pages are included in order with the exam. The exam is 8 pages, including the cover sheet.
- **Specific Intructions**
  - You will have **60 minutes** to complete this exam.
  - This exam is closed book, closed notes, and you may not use a calculator.
  - In order to receive full credit you must show your work. Include your computations on the exam paper.
  - Place a box around your final answer to each question when appropriate.

Total Possible Points	Score	Percent
100 (5 Extra Credit)		

1. (10 points) Consider the region bounded by $y = x^2$ and $y = \sqrt[3]{x}$ for $0 \leq x \leq 8$.

   (a) (2 points) Sketch the region bounded by the curves. Clearly label each curve and label any important points.

   (b) (8 points) Find the area of this region.

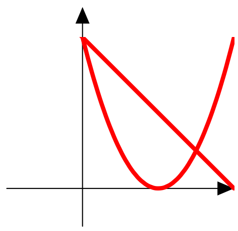
2. (8 points) In this problem you are going to find the number a such that the line $x = a$ divides the area under the curve $y = 1/x^2$ into two regions of equal area for $1 \leq x \leq 4$.

   (a) (2 points) Sketch a rough graph of the region and clearly indicate what you are looking for.

   (b) (6 points) Now, find the number a such that the line $x = a$ divdes the area under the curve $y = 1/x^2$ into two regions of equal area for $1 \leq x \leq 4$.

3. (7 points) The base of a solid is the region bounded by $y = 2 - x^2$ and the x-axis. Cross sections perpendicular to the y-axis are rectangles whose height is twice the length. Find the volume of this solid.

4. (20 points, 5 points each) Consider the region bounded by $y = x^2 - 4x + 4$, $y = 4 - x$, which is graphed below. Set up, but do not solve, an integral that finds the volume of this region when it is rotated about each of the following axes. **You do NOT need to simplify the integrand.** State the method that you are using. A sketch of the region has been provided for you.



(a) x-axis

(b) y-axis

(c) $x = -3$

(d) $y = 5$

5. (15 points) The temperature in a certain city (in °F) t hours after 8 AM was modeled by the function

$$T(t) = 5\sin\left(\frac{\pi t}{12}\right) - 20.$$

(a) (8 points) Use this equation to find the average temperature during the period from 8 AM to 8 PM. Give an exact answer with proper units AND then, using $\pi \approx 3$ give a rough estimate of what the average temperature is to the nearest whole number.

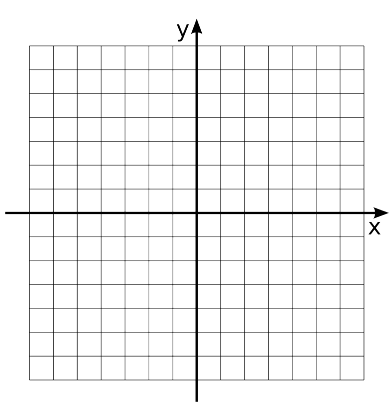
(b) (2 points) Explain why the Mean Value Theorem for integrals applies to the equation $T(t)$ on any interval $[a, b]$.

(c) (5 points) Find the time $t = c$ such that $T(c)$ is equal to the average value from part (a). Use the exact, not the approximate answer to do this. Give units with your answer.

6. (8 points) If the work required to stretch a spring 2 feet beyond its natural length is 24 ft-lbs, how much work is needed it stretch it one foot beyond its natural length? Give your final answer with proper units.

7. (7 points) Find the exact length of the curve $y = 1 + 6x^{3/2}$ for $0 \le x \le 1$.

8. (8 points) Find the surface area obtained by rotating the region $y = 4 - x^2$ between $x = 0$ and $x = 2$ about the y-axis. Begin by sketching the curve.



9. (15 points) Consider the region bounded by $y = \cos(2x)$, $x = 0$, and $y = 0$ on the interval $[0, \pi/8]$.

   (a) (5 points) Sketch curves on the interval from $[0, \pi]$, shade the region, and then find the area bounded by the curves.

   (b) (10 points) Find the centroid (also known as the center of mass) of this region.

10. (Extra Credit 5 points) A tank is full of water. Find the work required to pump the water out of the spout. Use 9.8 m/s^2 for the acceleration due to gravity and assume that water has a density of 1000 kg/m^3. (Note: Significant partial credit will be awarded for correct set-up and stating the correct units of the final answer.)