MATH253X-F01
Fall 2019

Final Exam

Name: ______________________

Instructions. (100 points) You have 120 minutes. Closed book, closed notes, no calculator. **The last page contains some unlabeled theorems from our course.** *Show all your work* to receive full credit.

(8$^{\text{pts}}$) **1.** Consider the points $A = (1, 2, -1)$, $B = (-3, 0, 1)$ and $C = (0, 3, 1)$.

(a) (3 pts) Give a parameterization of the straight line segment from A to B. Be sure you state what the parameter may range over.

(b) (5 pts) Find an equation (*not* a parameterization) for the plane containing A, B, C.

(6$^{\text{pts}}$) **2.** Sketch the region of integration of

$$\int_0^1 \int_{3y}^3 e^{(x^2)}\, dx\, dy.$$

Then use your sketch to reverse the order of integration and evaluate the integral.

(11pts) **3.** Assume a particle has velocity $\mathbf{v}(t) = (t+1)\mathbf{i} + 2\sqrt{t}\mathbf{j} + (t-1)\mathbf{k}$ for $t \geq 1$ with speed measured in m/s.

(a) (3 pts) Find the time(s) when acceleration is parallel to $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(b) (4 pts) Find the distance traveled from $t = 1$ s to $t = 3$ s.

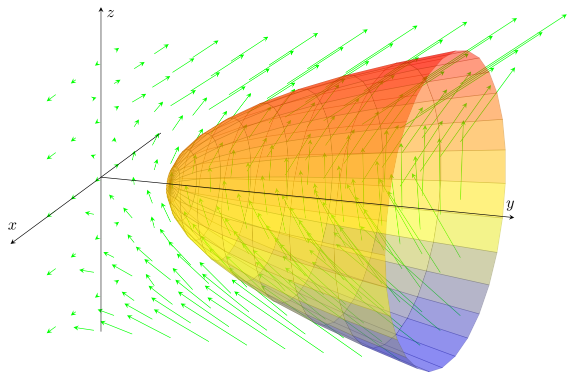
(c) (4 pts) Find the position vector $\mathbf{r}(t)$ at all times if $\mathbf{r}(1) = 2\mathbf{i} - \frac{1}{2}\mathbf{k}$.

(6pts) **4.** Use Lagrange multipliers to find the extreme values of the function $f(x, y) = x^2 - y^2$ along the parabola $x - y^2 = -1$.

(12pts) **5.** Consider the surface S parametrized by:

$$\mathbf{r}(u,v) = \left\langle u\cos v, u^2+1, u\sin v \right\rangle \quad \text{for} \quad 0 \le u \le 2\ ,\ 0 \le v \le 2\pi$$

in the vector field $\mathbf{F}(x,y,z) = \langle x, yz, y\rangle$ as illustrated below:



(a) (7 pts) Use Stokes' theorem to compute the circulation of $\mathbf{F}(x,y,z)$ around the oriented boundary curve C of the surface S NOT directly BUT as a surface integral using the given S.

(b) (5 pts) Find an equation of the tangent plane to the surface at the point $\left(\dfrac{1}{2}, 2, \dfrac{\sqrt{3}}{2}\right)$.

(8$^{\text{pts}}$) **6.** Sketch the two surfaces

$$x^2 + z^2 = 4, \quad y + z = 5$$

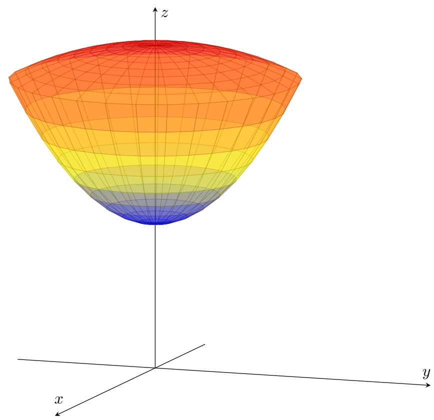
and highlight their curve of intersection. Then give a parameterization of that curve.

(8$^{\text{pts}}$) **7.** Find all critical points of the function

$$f(x, y) = x^3 - 6xy + 8y^3$$

and, to the extent possible, determine whether they are local maxima, local minima, or saddle points.

(9pts) **8.** Use **cylindrical coordinates** to find the mass of the solid enclosed below by the paraboloid $z = x^2 + y^2 + 1$ and above by the sphere $x^2 + y^2 + z^2 = 5$ if the density function is given by $\rho(x, y, z) = \dfrac{1}{z^2}$.



(8pts) **9.** Let S be the closed surface that encloses the eighth of the unit ball centered at the origin for which $x \geq 0$, $y \leq 0$ and $z \leq 0$, oriented outward. Use Gauss' Divergence Theorem and **spherical coordinates** to fully SET UP an integral computing the flux out of S of the vector field $\mathbf{F}(x, y, z) = \langle x^2, -2yx, xz\rangle$. DO NOT EVALUATE.

(8 pts) **10.** Consider the vector field

$$\mathbf{F}(x, y, z) = \langle e^{x-y} - z\sin(xz), z^2 - e^{x-y}, 2yz - x\sin(xz)\rangle.$$

(a) (5 pts) Find a potential function for $\mathbf{F}(x, y, z)$.

(b) (3 pts) Use your answer to part (a) to evaluate the work done by $\mathbf{F}$ if a particle follows a helical path from the point $(2, 0, 0)$ to the point $(2, 0, 1)$, spiraling counterclockwise one time around the z-axis.

(8 pts) **11.** Use Green's theorem to compute

$$\int_C \left(e^{\cos x} - x^2 y\right)\ dx + (\arctan y + xy)\ dy$$

over the closed curve C made up of the line segment from $(0, 0)$ to $(2, 0)$, then three quarters around the circle $x^2 + y^2 = 4$ until $(0, -2)$ then the line segment back to the origin.

(8pts) **12.** Let $f(x, y) = \dfrac{x}{y^2} + x^2 y$.

(a) (4 pts) What is the directional derivative of f at $(2, 1)$ when moving towards $(0, 2)$? What does it mean for function values?

(b) (4 pts) Let $x(s, t) = s^2 t$ and $y(s, t) = 2s - t$. Use the appropriate chain rule to find $\dfrac{\partial f}{\partial t}$ (no direct substitution). Your final answer should only contain s and t but DO NOT simplify.

Some formulas from theorems in the course:

$$f(B) - f(A) = \int_{AB} \nabla f \cdot d\mathbf{r}$$

$$\oint_{C=\partial R} P\,dx + Q\,dy = \iint_R Q_x - P_y\,dA$$

$$\oint_{C=\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

$$\oiint_{S=\partial V} \mathbf{F} \cdot d\mathbf{S} = \iiint_V \operatorname{div} \mathbf{F}\,dV$$