

Show all your work. Drawing pictures to help your reasoning is **strongly** encouraged.

1. (15 pts.) A metal plate of constant density is shaped like the region between the graphs of $y = x^2$ and $y = -x^2$ with $0 \leq x \leq 4$. Find the center of mass (\bar{x}, \bar{y}) of the plate. You may use the symmetry of the region to reduce the amount of calculation you need to do.

2. (13 pts.) An elliptical window is to be located in a vertical wall of an underwater observatory. If the window has boundary given by the equation

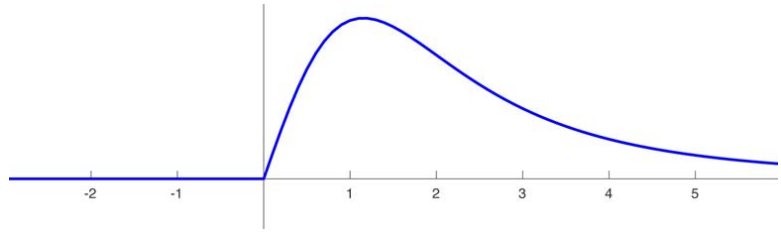
$$x^2 + 4y^2 = 4$$

where x, y are measured in feet, and the surface of the water and is located at $y = 20$, find the force on the window due to the water. (The weight density of water is 62.4 lbs/ft^3 .) Give your answer as an integral, but **do not evaluate** it. **Specify units** for your answer

3. The length t , in minutes, of a telephone call placed through a certain company has probability density

$$f(t) = \begin{cases} 0 & \text{if } t < 0, \\ A \frac{t}{(4+t^2)^2} & \text{for } t \geq 0 \end{cases}$$

where $A > 0$ is some constant, and the graph of f is shown.



- (a) (6 pts.) Determine the value of A .
- (b) (8 pts.) Find the probability that a call lasts less than 3 minutes. (Use the value of A you found in part(a); if you did not find a value for A , you may assume $A = 7$.)
- (c) (4 pts.) Give an integral which computes the mean length of calls placed through the company. **Do not evaluate** your integral.

4. (a) (10 pts.) Compute the volume of the object obtained by rotating the region between $y = x^3$ and the x -axis for $1 \leq x \leq 3$ about the horizontal axis $y = -1$.

(b) (10 pts.) Give an integral to compute the surface area of the object obtained by rotating the graph of $y = x^3$, $1 \leq x \leq 3$ about the axis $y = -1$. **Do not evaluate** your integral.

5. (13 pts.) A 180 lb person is attached to the bottom of a rope that hangs 200 ft over a cliff. The rope weighs 0.25 lbs/ft. How much work will be done in pulling the person and rope up to the top of the cliff? **Specify units** for your answer.

6. (8 pts.) The temperature, in $^{\circ}C$, over a 24 hour period is given by the function

$$f(t) = 18 + t/3 + 3 \sin(\pi t/12), \quad 0 \leq t \leq 24.$$

What is the average temperature over that time? **Specify units** for your answer.

7. (13 pts.) Find the volume of the object obtained by rotating the region between $y = 8 - x^2$ and $y = x^2$ about the vertical axis $x = 3$.