Math 252 Exam 1
09/21/2020

Name:____________________________
Instructor (circle one): Rhodes Alam

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\sin 2x = 2\sin x \cos x$
$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$	$\cos 2x = \cos^2 x - \sin^2 x$

1. (65 pts. – 13 pts. each) Evaluate the following indefinite integrals, showing your work.

(a) $\displaystyle\int \frac{1}{(x^2 - 9)^{3/2}}\, dx$

(b) $\displaystyle\int_0^1 \arctan x \, dx$

(c) $\displaystyle\int \sec^3 2x \tan^3 2x \, dx$

(d) $\displaystyle\int \frac{2x^2 - 2x - 2}{x^3 - x} \, dx$

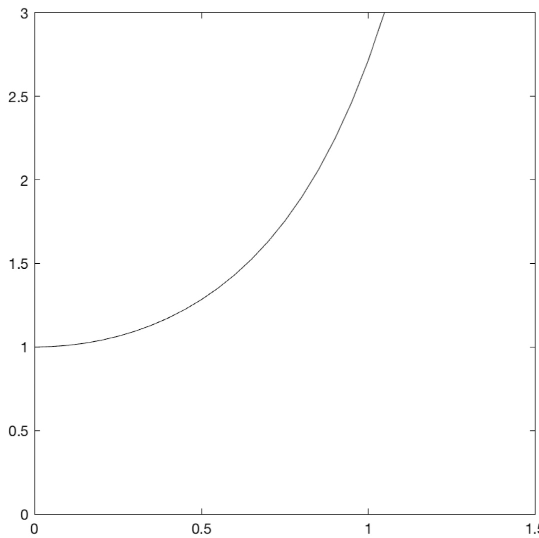
(e) $\displaystyle\int_3^7 x\sqrt{x-3}\,dx$

2. (20 pts. – 10 pts. each) Determine whether the following improper integrals converge or diverge. If they converge, compute their values.

   (a) $\displaystyle\int_0^\infty \frac{1}{(1+x)^{5/2}}\,dx$

   (b) $\displaystyle\int_0^{\pi/2} \tan\theta\,d\theta$

3. (15 pts.) The function $f(x) = e^{(x^2)}$ is graphed.



(a) (2 pts.) On the graph, sketch the rectangles whose areas would form the midpoint sum M_4 approximating the integral $\int_0^1 e^{(x^2)}\,dx$.

(b) (5 pts.) Write out the midpoint sum M_4, using numbers in all terms. **Do not simplify.**

..

(c) (3 pts.) Does M_4 give too large or too small an estimate? Briefly explain how you can tell.

(d) (5 pts.) The error bound for a midpoint sum is

$$|\text{Error}(M_n)| \leq \frac{K(b-a)^3}{24n^2},$$

where $|f''(x)| \leq K$ on $[a, b]$. Use this to determine n so that the error in approximating $\int_0^1 e^{(x^2)}\,dx$ is no larger than 0.0001.